## Section 3.3: Reduced echelon form

New vocabulary:

- reduced echelon matrix
- Homogeneous system of equations
- Trivial solution (to a HSOE) = Zero solution · ?]=leading

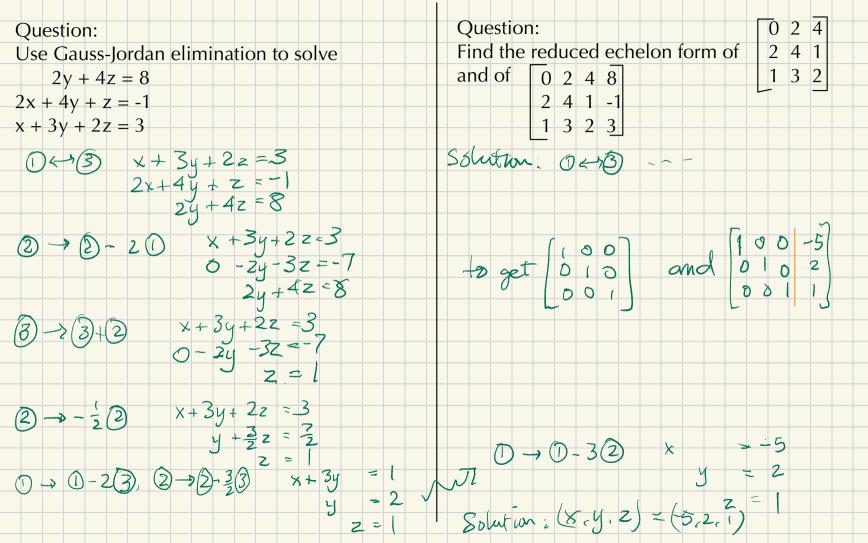
diagonal

- Principal diagonal
- Identity matrix \_\_\_\_\_ ?
- Gauss-Jordan elimination

the right hand side is all D

Rene de madix: 00+ + × - - - × 0 1. Zevo nows at the pottom 2. Each leading entry occurs in a later column Than the reading entres of eacher rows reduced echelon matrix hal 1. 2. and 3. Leading entres are 1, and all other entries in a column with a l.e. are O × 001\*0\* 00001\* ×  $\mathcal{O}$ 0

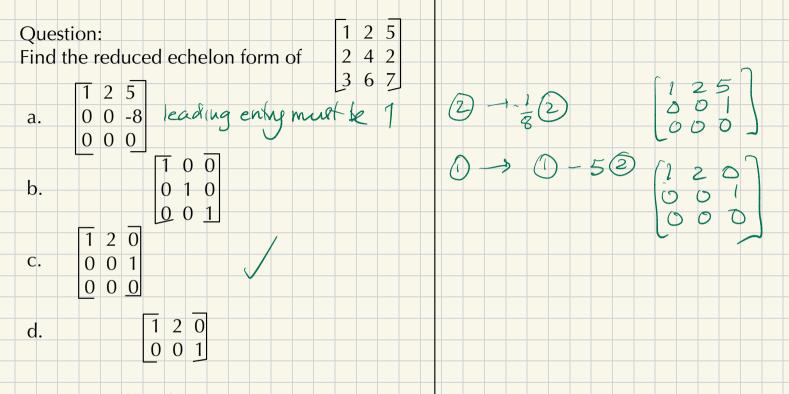
Recall: leading entries, free variables



## Theorems

- Reduced echelon form is unique. (Theorem 1)
- More variables than equations imply the number of solutions is either 0 or infinity. (Theorem 3+)
- A homogeneous system always has a solution.
- A system with n variables and n equations has a unique solution if and only if the reduced echelon form to the left of the line is the identity.
- If there are infinitely many solutions, then there are free variables
- A homogeneous system with n variables and n equations has a unique solution if and only if the coefficient matrix has reduced echelon form the identity. (Theorem 4)

- vanaple m IF n >m there is i IF n >m, there is always a free variable if the system is consistent, we get infinitely many solutions, inconsistent: no solutions, The zero solution = the trivial solution The identity on the left produces a unique solution /.
- If we don't get the identity on the left there is a row of zeros and a free 1 L'variable.



e. None of the above

## True or false?

- . If a system of linear equations has more equations than unknown variables then there is no solution.
- 2. If a system has fewer equations than variables there is always a solution.
- 3. If a system has fewer equations than variables and there is at least one solution then there are infinitely many solutions.